**Roll : 240111**

**Non-Exact Differential Equation**

**Definition of Non-Exact Differential Equation:**

A **non-exact differential equation** is a first-order equation of the form:

which **does not satisfy** the exactness condition:

**✅ How to Check Exactness:**

Given:

Check:

If they are not equal, the equation is **non-exact**.

**Now Examples From The Book Of S.L. Ross-**

**🔹 Example 1 (Non-exact):**

**Equation:**

Let:  
   

Now :  
   

As ,

⇒ This is a **non-exact** equation.

**🔹 Example 2 (Non-exact)**

**Equation:**  
  
  
**Check:**  
    ,

As ,

⇒ This is a **non-exact** equation.

**🔹 Example 3 (Non-exact)**

**Equation:**

**Check:**  
    ,

As

⇒ This is a **non-exact** equation.

**🔹 Example 4 (Non-exact)**

**Equation:**

**Check:**  
    ,

As

⇒ This is a **non-exact** equation.

**🔹 Example 5 (Non-exact)**

**Equation:**

**Check:**  
    ,

As

⇒ This is a **non-exact** equation.

**🔹 Example 6 (Non-exact)**

**Equation:**

**Check:**  
    ,

As

⇒ This is a **non-exact** equation.

**🔹 Example 7 (Non-exact)**

**Equation:**

**Check:**  
    ,

As

⇒ This is a **non-exact** equation.

**Roll : 240112**

**Exact D. E.**

Exact differential equation

Method of Grouping Standard Grouping

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The expression {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} is exact differential if there exists a function such that

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mi>d</mml:mi><mml:mi>u</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

If the expression  
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} is exact differential then the equation

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

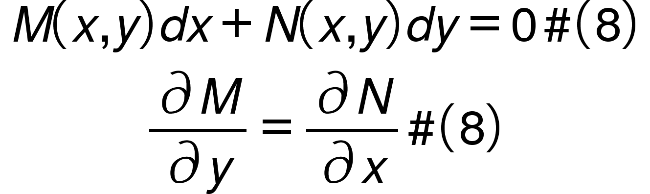
exact differential as equation

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mo>&#x2234;</mml:mo><mml:mi>d</mml:mi><mml:mi>u</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Integrating, we get

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>u</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mo>=</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>0</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
This is the general solution the  
exact Differential equation

Theorem: state and preve the necessary and sufficient condition for exactness of a differential equation:  
or  
Find the Condition {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} may be exact differential equation.  
Answer: statement  
The necessary and the sufficient condition for exactance of the differential equation

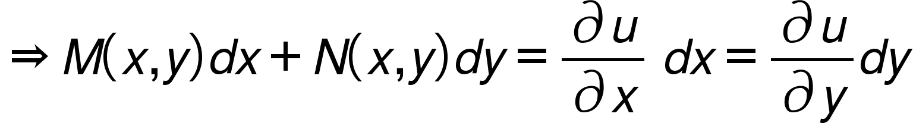


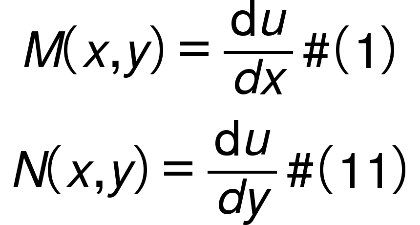
Proof: part-1 we assume that the defferential equation

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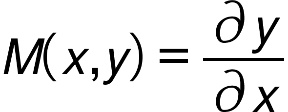
Then there exists a function such that

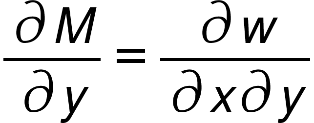
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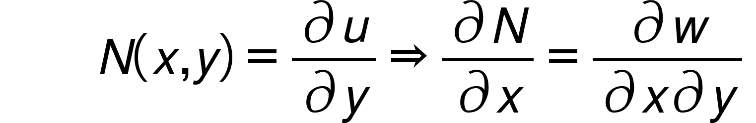
 ccomparing on both sides get,

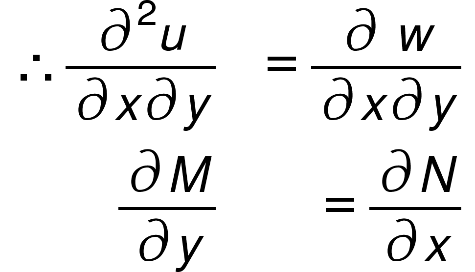


From (i)



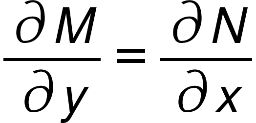


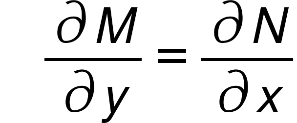
Frem (2)



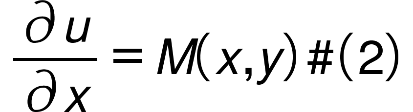
which is the necessary Condition.

(part-II) the Sufficient Condition:

Given , we have to show that  
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We assume that there exists a Function {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>u</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} such that



Integrating (2) by paratially w, n to , we get

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mtable columnalign=\"right\"><mml:mtr><mml:mtd/><mml:mtd><mml:mi>&#xA0;</mml:mi><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>u</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:mfrac><mml:mo>=</mml:mo><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mi>M</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>u</mml:mi><mml:mo>=</mml:mo><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mi>M</mml:mi><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>&#x3D5;</mml:mi><mml:mo>(</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mtable columnalign=\"right\"><mml:mtr><mml:mtd><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>u</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd><mml:mtd><mml:mi>&#xA0;</mml:mi><mml:mo>=</mml:mo><mml:mfrac><mml:mo>&#x2202;</mml:mo><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mi>M</mml:mi><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>&#x3D5;</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>&#xA0;</mml:mi><mml:mo>=</mml:mo><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>M</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>&#x3D5;</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>&#xA0;</mml:mi><mml:mo>=</mml:mo><mml:mo>&#x222B;</mml:mo><mml:mi>&#xA0;</mml:mi><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>N</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:mfrac><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>&#x3D5;</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>&#xA0;</mml:mi><mml:mo>=</mml:mo><mml:mi>N</mml:mi><mml:mo>+</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>&#x3D5;</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr></mml:mtable></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>N</mml:mi><mml:mo>=</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>u</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac><mml:mo>-</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>&#x3D5;</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr><mml:mtr><mml:mtd/><mml:mtd><mml:mi>M</mml:mi><mml:mo>=</mml:mo><mml:mfrac><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>u</mml:mi></mml:mrow><mml:mrow><mml:mo>&#x2202;</mml:mo><mml:mi>x</mml:mi></mml:mrow></mml:mfrac></mml:mtd></mml:mtr></mml:mtable></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Now,

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So that {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:msub><mml:mi>M</mml:mi><mml:mi mathvariant=\"normal\">&#xA0;</mml:mi></mml:msub><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>N</mml:mi><mml:mo>(</mml:mo><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi><mml:mo>)</mml:mo><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"} y is an exact D. E.

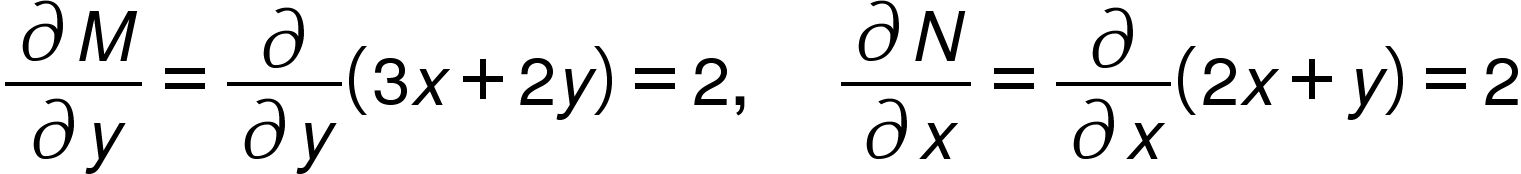
Exact Differential Equations Exercise

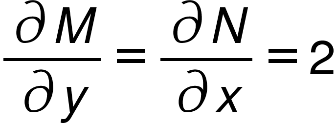
# Exercise 1: {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>3</mml:mn><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 1: Verify exactness**  
Given:

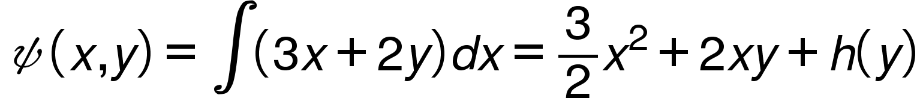
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Compute partial derivatives:

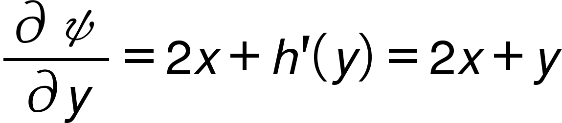


Since , the equation is exact.

**Step 2: Find potential function**   
Integrate with respect to :

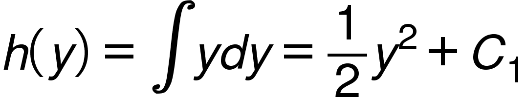


**Step 3: Determine**   
Differentiate with respect to and equate to :

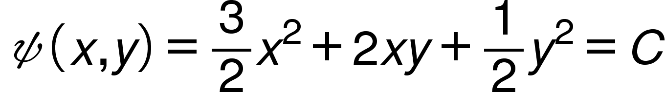


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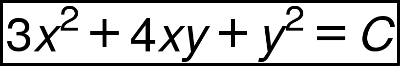
Integrate with respect to :



**Step 4: Write general solution**



Multiply by 2 to simplify:

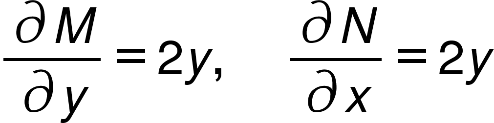


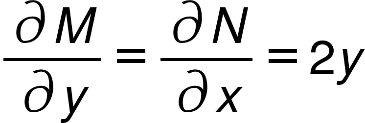
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**Step 1: Verify exactness**  
Given:

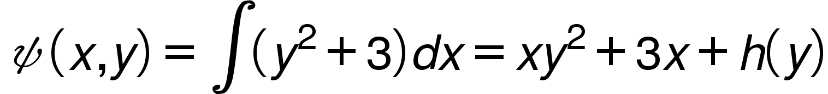
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Compute partial derivatives:

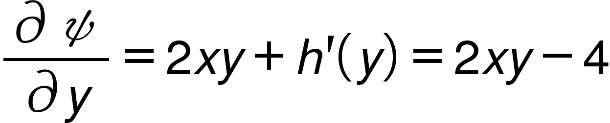


Since , the equation is exact.

**Step 2: Find potential function** {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>&#x3C8;</mml:mi><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
Integrate with respect to :



**Step 3: Determine** {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
Differentiate with respect to and equate to :

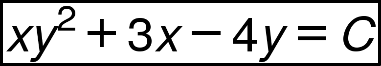


{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>4</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Integrate:

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>4</mml:mn><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>1</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 4: Write general solution**



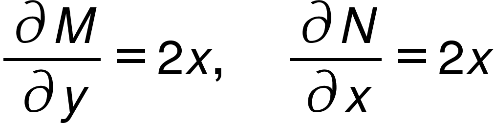
# 

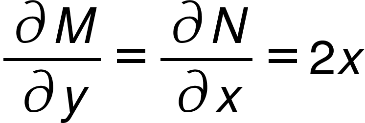
# Exercise 3: {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:mn>1</mml:mn></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>4</mml:mn><mml:mi>y</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 1: Verify exactness**  
Given:

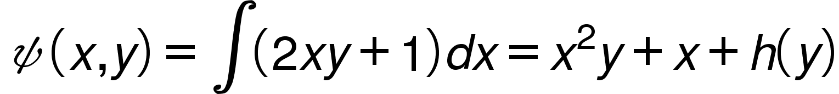
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:mn>1</mml:mn><mml:mo>,</mml:mo><mml:mo>&#x2001;</mml:mo><mml:mi>N</mml:mi><mml:mo>=</mml:mo><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>4</mml:mn><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Compute partial derivatives:

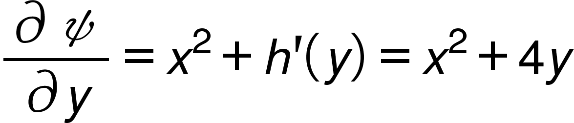


Since , the equation is exact.

**Step 2: Find potential function** {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>&#x3C8;</mml:mi><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
Integrate with respect to :



**Step 3: Determine**   
Differentiate with respect to and equate to :

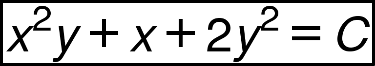


{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mn>4</mml:mn><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

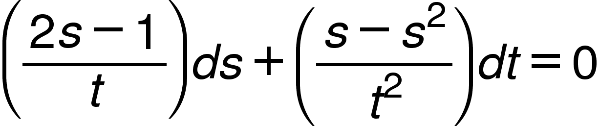
Integrate:

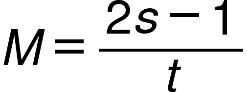
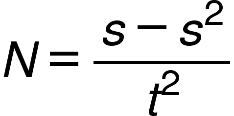
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:msup><mml:mi>y</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>1</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 4: Write general solution**

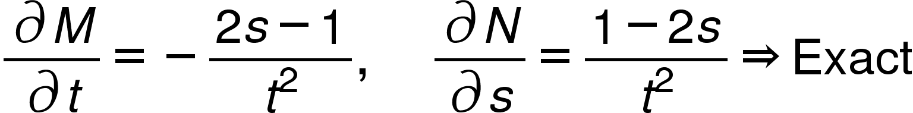


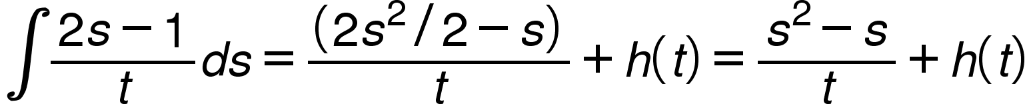
# Exercise: 4

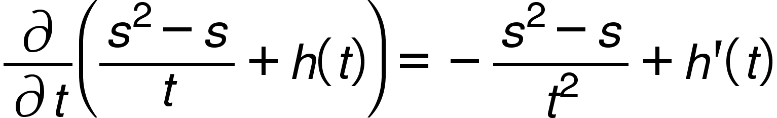


, 

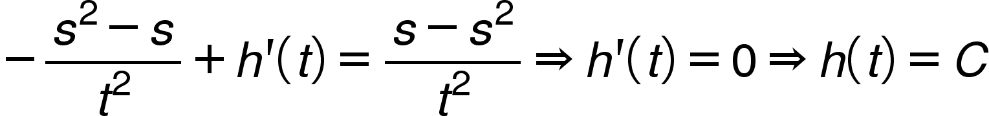
Check exactness:



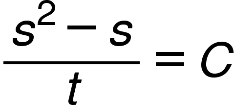




Set equal to :



Solution:

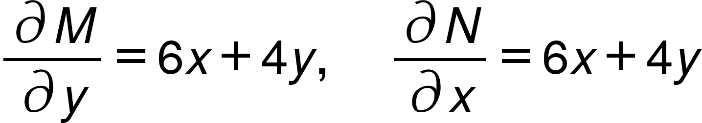


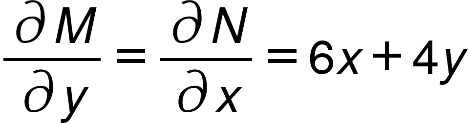
# Exercise 5: {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>6</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:msup><mml:mi>y</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>5</mml:mn></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mn>3</mml:mn><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>4</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>-</mml:mo><mml:mn>6</mml:mn></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 1: Verify exactness**  
Given:

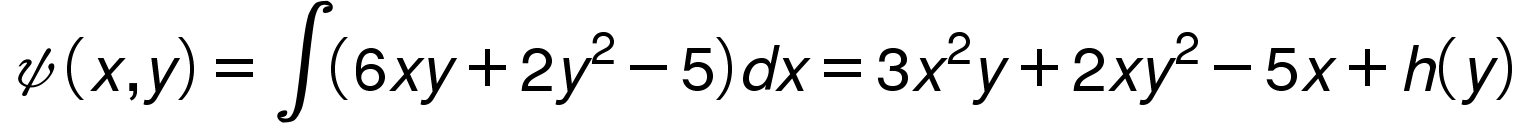
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>=</mml:mo><mml:mn>6</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:msup><mml:mi>y</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>-</mml:mo><mml:mn>5</mml:mn><mml:mo>,</mml:mo><mml:mo>&#x2001;</mml:mo><mml:mi>N</mml:mi><mml:mo>=</mml:mo><mml:mn>3</mml:mn><mml:msup><mml:mi>x</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>4</mml:mn><mml:mi>x</mml:mi><mml:mi>y</mml:mi><mml:mo>-</mml:mo><mml:mn>6</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Compute partial derivatives:

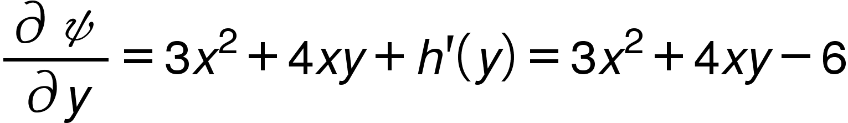


Since , the equation is exact.

**Step 2: Find potential function** {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>&#x3C8;</mml:mi><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
Integrate with respect to :



**Step 3: Determine**   
Differentiate with respect to and equate to :

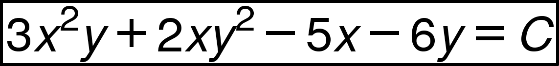


{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>6</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Integrate:

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mo>-</mml:mo><mml:mn>6</mml:mn><mml:mi>y</mml:mi><mml:mo>+</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>1</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 4: Write general solution**



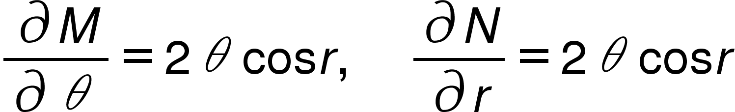
# 

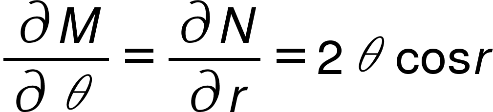
# Exercise 6: {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:msup><mml:mi>&#x3B8;</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>1</mml:mn></mml:mrow></mml:mfenced><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>r</mml:mi><mml:mi>d</mml:mi><mml:mi>r</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>&#x3B8;</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>r</mml:mi><mml:mi>d</mml:mi><mml:mi>&#x3B8;</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 1: Verify exactness**  
Given:

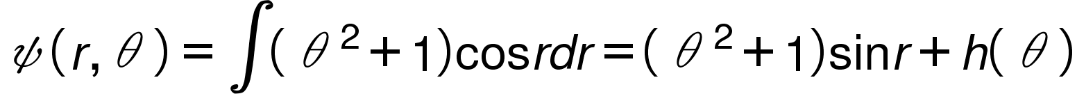
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>=</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:msup><mml:mi>&#x3B8;</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:mn>1</mml:mn></mml:mrow></mml:mfenced><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi mathvariant=\"normal\">o</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi>r</mml:mi><mml:mo>,</mml:mo><mml:mo>&#x2001;</mml:mo><mml:mi>N</mml:mi><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:mi>&#x3B8;</mml:mi><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">i</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>r</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Compute partial derivatives:

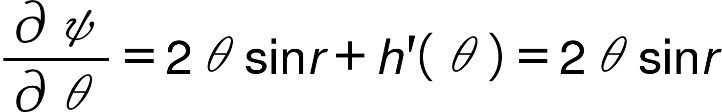


Since , the equation is exact.

**Step 2: Find potential function**   
Integrate with respect to :



**Step 3: Determine**   
Differentiate with respect to and equate to :

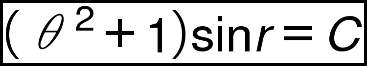


{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mfenced separators=\"|\"><mml:mi>&#x3B8;</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Integrate:

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>&#x3B8;</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>1</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 4: Write general solution**

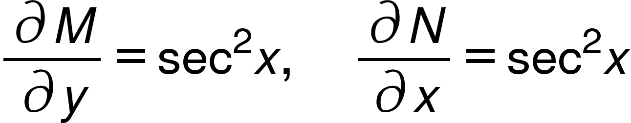


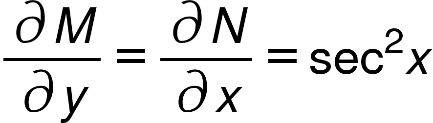
# Exercise 7: {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>y</mml:mi><mml:msup><mml:mrow><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">e</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi></mml:mrow><mml:mn>2</mml:mn></mml:msup><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">e</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi>x</mml:mi><mml:mi mathvariant=\"normal\">t</mml:mi><mml:mi mathvariant=\"normal\">a</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mfenced separators=\"|\"><mml:mrow><mml:mi mathvariant=\"normal\">t</mml:mi><mml:mi mathvariant=\"normal\">a</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi></mml:mrow></mml:mfenced><mml:mi>d</mml:mi><mml:mi>y</mml:mi><mml:mo>=</mml:mo><mml:mn>0</mml:mn></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 1: Verify exactness**  
Given:

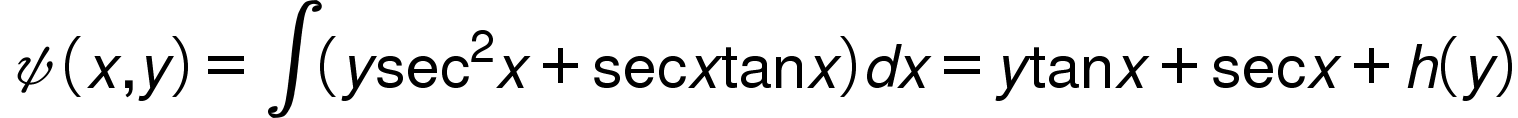
{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>M</mml:mi><mml:mo>=</mml:mo><mml:mi>y</mml:mi><mml:msup><mml:mrow><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">e</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi></mml:mrow><mml:mn>2</mml:mn></mml:msup><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mi mathvariant=\"normal\">s</mml:mi><mml:mi mathvariant=\"normal\">e</mml:mi><mml:mi mathvariant=\"normal\">c</mml:mi><mml:mi>x</mml:mi><mml:mi mathvariant=\"normal\">t</mml:mi><mml:mi mathvariant=\"normal\">a</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mo>&#x2001;</mml:mo><mml:mi>N</mml:mi><mml:mo>=</mml:mo><mml:mi mathvariant=\"normal\">t</mml:mi><mml:mi mathvariant=\"normal\">a</mml:mi><mml:mi mathvariant=\"normal\">n</mml:mi><mml:mi>x</mml:mi><mml:mo>+</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

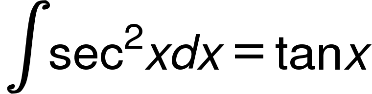
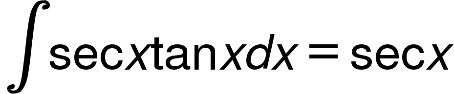
Compute partial derivatives:



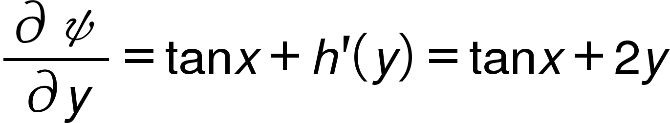
Since , the equation is exact.

**Step 2: Find potential function** {"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>&#x3C8;</mml:mi><mml:mfenced separators=\"|\"><mml:mrow><mml:mi>x</mml:mi><mml:mo>,</mml:mo><mml:mi>y</mml:mi></mml:mrow></mml:mfenced></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}  
Integrate with respect to :



Note: , 

**Step 3: Determine**   
Differentiate with respect to and equate to :



{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mi mathvariant=\"normal\">'</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:mn>2</mml:mn><mml:mi>y</mml:mi></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

Integrate:

{"mathml":"<mml:math style=\"font-family:null;font-size:null;\" xmlns:m=\"http://schemas.openxmlformats.org/officeDocument/2006/math\" xmlns:mml=\"http://www.w3.org/1998/Math/MathML\"><mml:mi>h</mml:mi><mml:mfenced separators=\"|\"><mml:mi>y</mml:mi></mml:mfenced><mml:mo>=</mml:mo><mml:msup><mml:mi>y</mml:mi><mml:mn>2</mml:mn></mml:msup><mml:mo>+</mml:mo><mml:msub><mml:mi>C</mml:mi><mml:mn>1</mml:mn></mml:msub></mml:math>","origin":"MathType Legacy","version":"v3.18.2"}

**Step 4: Write general solution**

